Modern Portfolio Theory with Particle Swarm Optimizer

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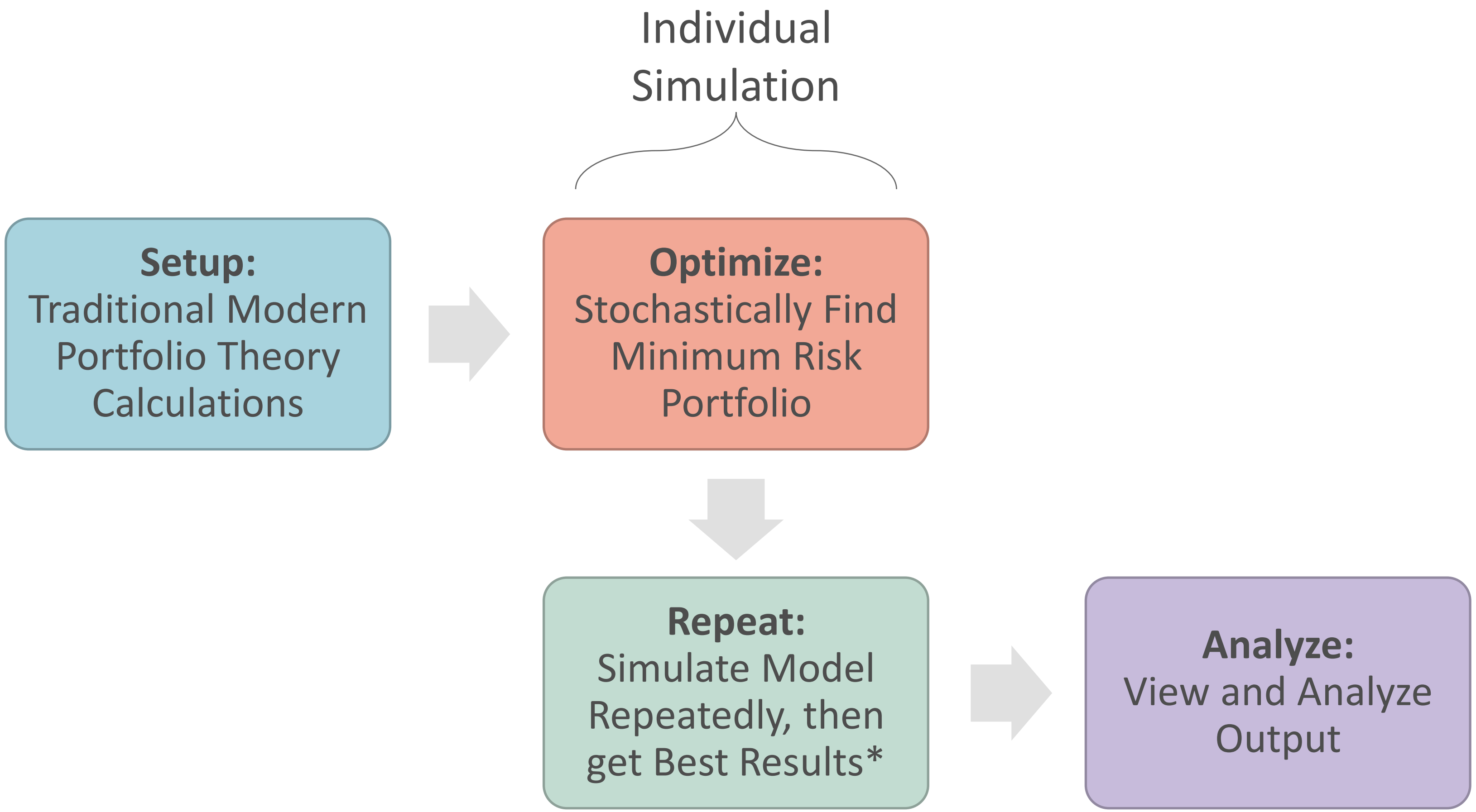
Overview of Modeling



Model Formulation:
Modern Portfolio Theory

Modern Portfolio Theory (MPT) Overview

Pull Stock Data from Yahoo Finance (`yfinance`)

Prepare data for portfolio choice modeling, including Nominal / Expected / Excess Returns, Variance-Covariance Matrix

Model framework is structurally similar to tribal-liquidity funds, and can easily transition calculation components

Model Formulation: Particle Swarm Optimizer

Minimize the Risk of Portfolio

- Decision Variable: Weights to invest into each stock
- Risk Defined as

$$risk = \sqrt{Weights.VarCov.Weights^T}$$

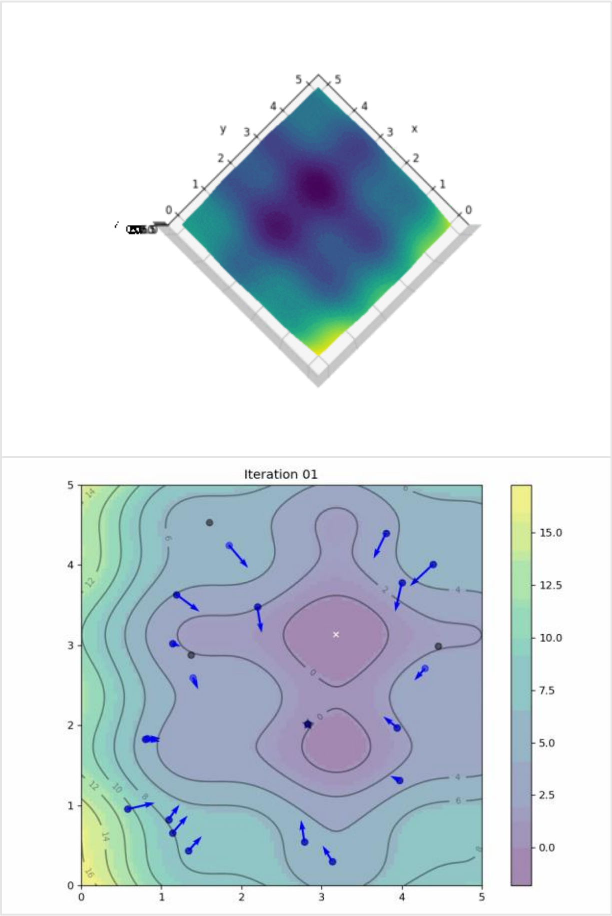
- Optimization Problem:

Minimize: $risk$
Subject to:
$\sum_{stock \in StockList} Weights_{stock} = 1$
$\sum_{stock \in StockList} (Weights_{stock} \times ExpectedReturns_{stock}) \geq minDesiredReturn$

This calculation is for annual returns *

PSO Overcomes Non-Linear Objectives

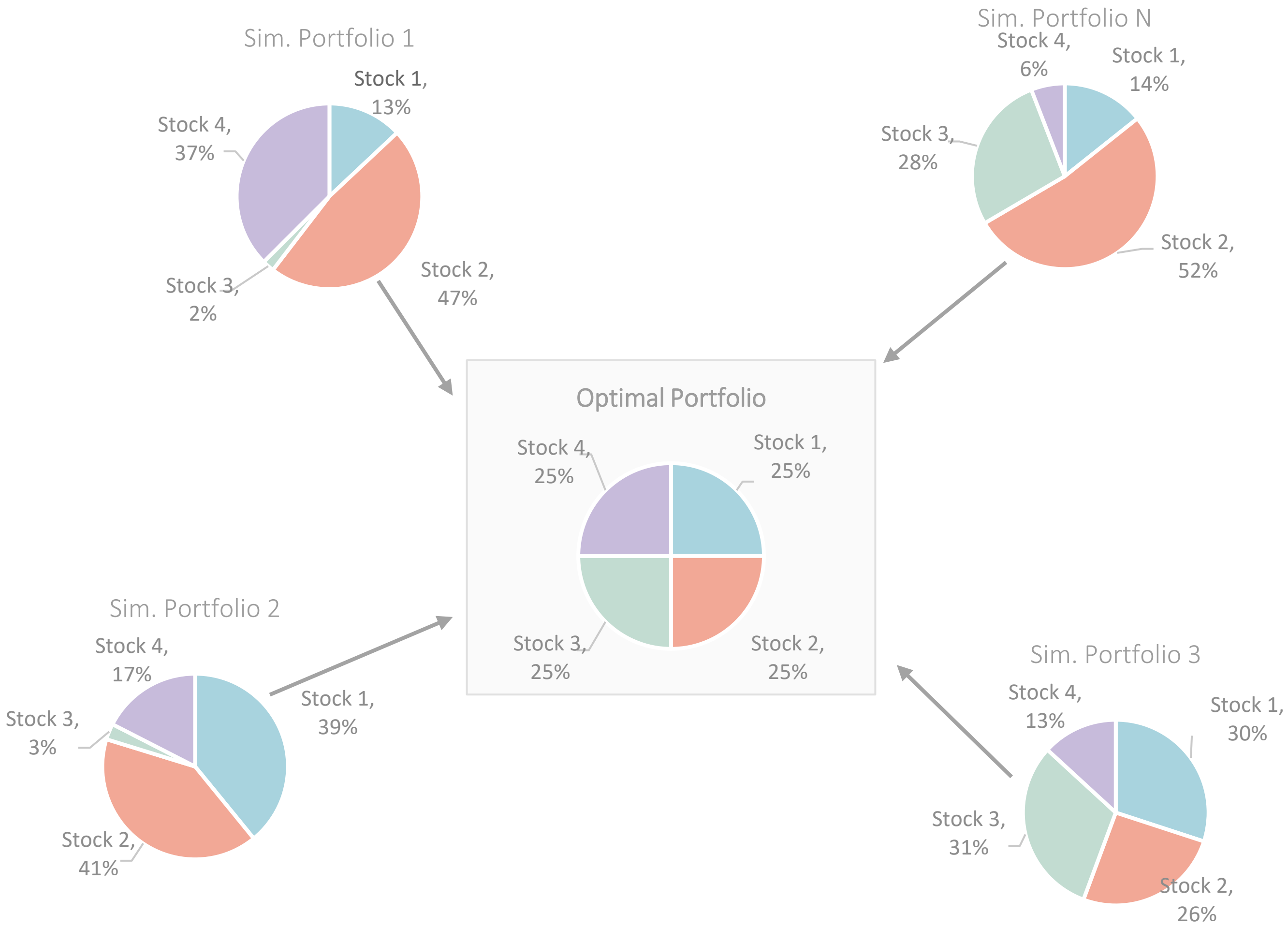
1. Metaheuristic algorithm that emulates organisms, or "particles" moving in swarms
2. Helps overcome local minima for non-linear modeling
3. Basic idea:
   - Swarm of particles starting random locations
   - Each particle has no understanding of where the overall swarm is headed
   - Each particle only knows where their immediate surrounding environment is headed
   - Collectively will arrive at final set of weights
   - *Repeat* process **n** times to better overcome local minima



Above model animation adapted from *Machine Learning Mastery*

PSO Used to Minimize Risk of Portfolio

1. Minimizing Risk involves non-linear objective
2. Use PSO algorithm to randomly adjust swarm of stock weights to find the minimum risk portfolio*
3. *Repeat* process **n** times to better overcome local minima



PSO Velocity and Position update equation used for modeling *

PSO Conceptual Process Overview

Initialize

- Stochastically initialize Swarm of portfolios (or particles) with feasible solutions

Update

- Randomly change the weights based on PSO velocity update equation

Evaluate

- Evaluate the risk and expected return of all portfolios in swarm

Document Best

- Document the best evaluated Weights
- Can either use global or local neighborhood

Stop Algorithm

- Algorithm stops when reached number of desired iterations, and
- Is feasible

Individual Simulation Repeated

PSO Velocity Update Equation

- Answer the question: how to determine the "movement" of the portfolios, or particles, upon each iteration

$$V_i^{t+1} = \underbrace{V_i^t}_{\text{Inertia}} + \underbrace{\varphi_1 . r_1 (P_i - X_i^t)}_{\text{Cognitive Component}} + \underbrace{\varphi_2 . r_2 (P_g - X_i^t)}_{\text{Social Component}}$$

where $r_1, r_2 \sim U(0,1)$

and acceleration constants φ_1, φ_2

- **Position Update:** $X_i^{t+1} = X_i^t + V_i^{t+1}$

PSO Velocity Update Equation

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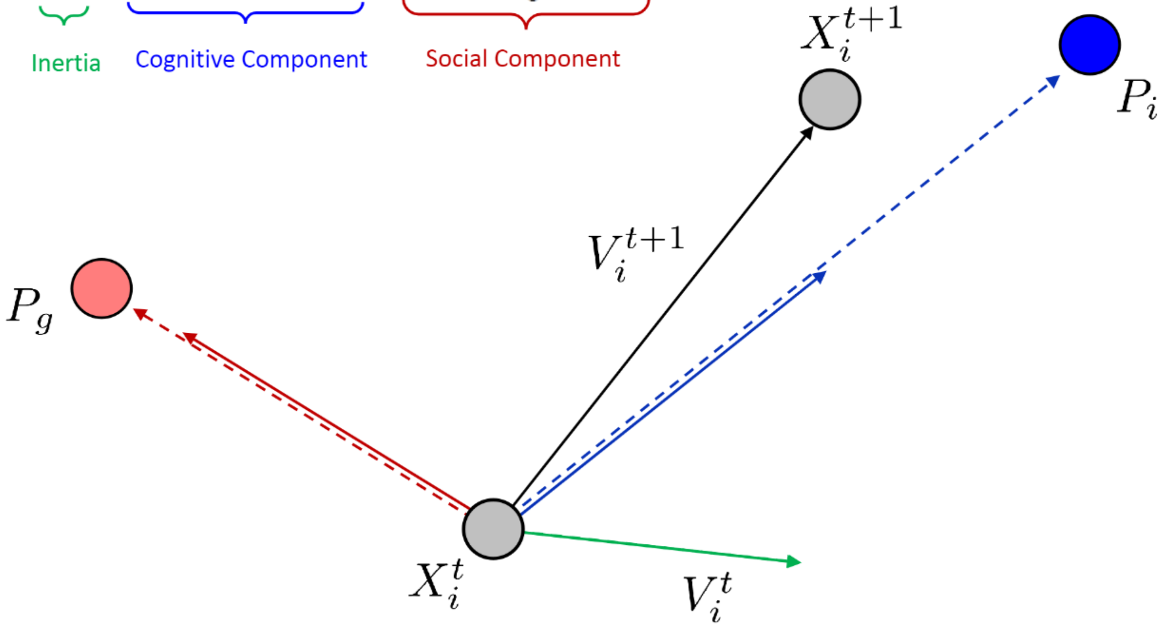
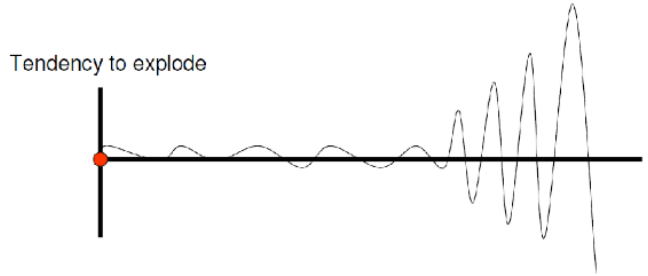


PSO Velocity Update Parameters

- Parameters that adjust size of movement

- Acceleration constants: φ_1, φ_2
  - small values limit the movement of the particles
  - large values : tendency to explode toward infinity
  - In general,

$$\varphi_1 + \varphi_2 \leq 4$$



- Maximum velocity

$$\text{If } v_{ij} > v_{\max} \text{ then } v_{ij} = v_{\max}$$
$$\text{else if } v_{ij} < -v_{\max} \text{ then } v_{ij} = -v_{\max}$$

PSO Velocity Update Parameters

- Parameters that adjust size of movement

  - Inertia weight:

$$V_i^{t+1} = w.V_i^t + \varphi_1.r_1(P_i - X_i^t) + \varphi_2.r_2(P_g - X_i^t)$$

  - Scales the previous velocity
  - Control search behavior
    - High values → exploration
    - Low values → exploitation

Summary

PSO SWOT Analysis

1

STRENGTHS

Time complexity while increasing investable assets
Highly flexible in portfolio choice modeling
Tends to expose in many assets

2

WEAKNESSES

Stochastic yields differing results each time

SWOT

3

OPPORTUNITIES

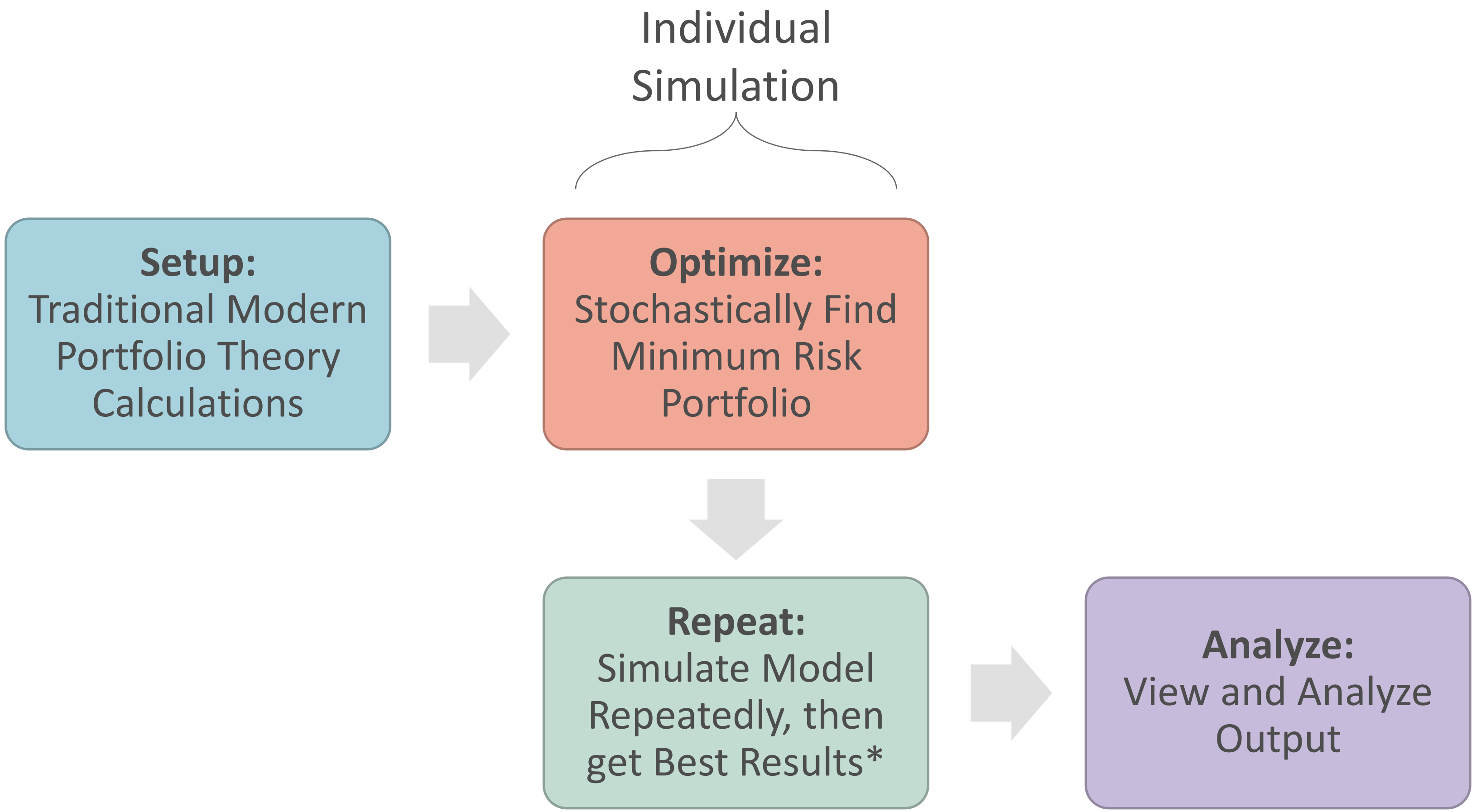
Easily can enhance or change modeling approach
Easily integrate machine learning into portfolio choice evaluation

4

THREAT

Could be seen as "black-box" modeling without proper communication

Summary


Individual Simulation
Setup:
Traditional Modern Portfolio Theory Calculations
Optimize:
Stochastically Find Minimum Risk Portfolio
Repeat:
Simulate Model Repeatedly, then get Best Results*
Analyze:
View and Analyze Output